Spatial Transformations in Graph Comprehension

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Introduction

Although it is apparent that people are able to make inferences from graphs, it is presently unclear *how* they do so, even from simple graphs. Current theories of graph comprehension are largely silent about the processes by which such inferences are made (e.g., Freedman & Shah, 2002; Pinker, 1990). We propose that people use spatial reasoning, in the form of spatial transformations (Trafton, Trickett, & Mintz, in press), to answer inferential questions. Spatial transformations are cognitive operations that a person performs on internal or external visualizations, such as graphs. They occur when people must *mentally* create or delete something (e.g., a line) on the image in order to facilitate problem solving, and may be related to hypothetical drawing (Shimojima & Fukaya, 2003). This paper investigates the use of spatial transformations when people need to make inferences from graphs.

Method

Eight GMU undergraduates participated for course credit. Participants were shown 30 unlabelled line graphs and asked for the value of the y axis at a given point on the x axis. This point on the x axis was indicated by a red arrow in one of three different positions, creating three conditions: read-off (arrow beneath line), near (arrow slightly beyond line), and far (far beyond end of line) (see Figure 1 for examples). Participants were shown 10 of each graph/condition combination, in random order; performance was self-paced, with a blank screen appearing between each graph.

To answer the question in the read-off condition participants had to move their eyes in perpendicular fashion from the red arrow to intersect the line, move their eyes from that intersection to the y axis, and read off the appropriate value. No spatial transformations were required for this task. To answer the question in the near and far conditions, we propose that participants would mentally extend the line prior to locating its intersection with the perpendicular from the red arrow and completing the task. The extension—a mental manipulation—constitutes a spatial transformation. Spatial transformation theory predicts that the longer the extension, the longer it takes; thus, we predict that participants will be fastest in the read-off (no extension) condition, somewhat slower in the near (shorter extension) condition, and slowest in the far (longer extension) condition. We also predict that accuracy will decrease with
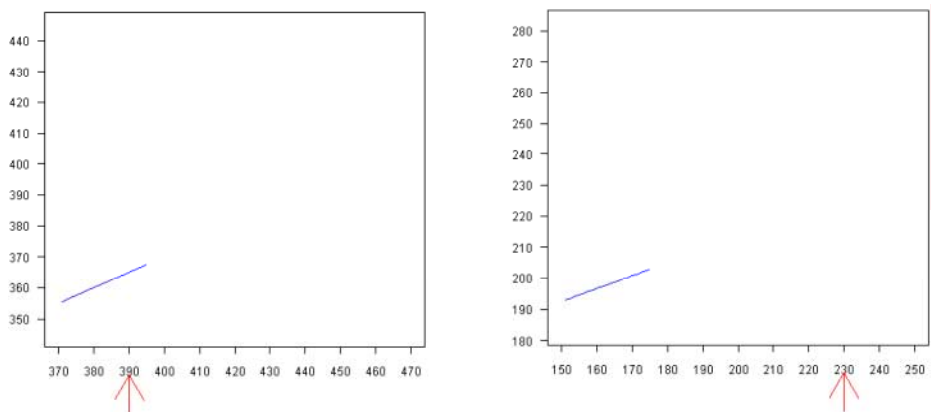
Fig. 1. Line graph read-off condition (left) and far condition (right). In the near condition (not illustrated) the arrow was 15 units to the right of the end of the line, along the x axis.

increased use of spatial transformations, as people must move further from "anchor points" on the graph to obtain needed information. Thus, we predict that participants will be most accurate in the read-off condition, somewhat less accurate in the near condition, and least accurate in the far condition.

Results and Discussion

We measured accuracy as the absolute value of the participant's response minus the correct response. A score of 0 thus means the answer was completely accurate; increased scores represent a *decrease* in accuracy. Response times (RT) represent the amount of time between graph presentation and entry of the participant's response. Responses with an accuracy score of more than 100 were considered outliers and excluded from analysis, as were responses whose RT was greater than 3 standard deviations above the mean. Outliers constituted less than 5% of the data.
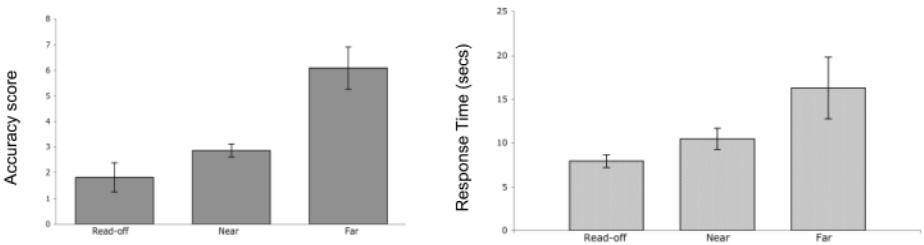

Fig. 2a (left). Mean accuracy scores for the read-off, near, and far tasks
Fig. 2b (right). Mean response times in seconds for the read-off, near, and far tasks

As Fig. 2a shows, participants were most accurate on the read-off task, less accurate on the near task, and least accurate on the far task, repeated measures ANOVA $F(2, 14) = 15.46$, $p < .01$, linear trend $F(1, 7) = 18.98$, $p < .01$. These results are consistent with our hypothesis that participants use spatial transformations to execute the inference tasks. The read-off task required no spatial transformations, but as participants mentally extended the line to find a hypothetical point of intersection with the arrow, the point of intersection became increasingly distant from the "anchor" of the y axis. Participants were decreasingly accurate as a result.

It is possible that participants' engaged in a speed-accuracy tradeoff; however, the response time data indicate that they became *slower* as they became *less* accurate. The response time data also point to the use of spatial transformations. Participants were fastest on the read-off task, slower in the near task, and slowest on the far task, $F(2, 14) = 4.93$, $p < .05$, linear trend $F(1, 7) = 6.7$, $p < .05$ (see Figure 2b). The linear trend is consistent with the idea that a longer extension takes more time to execute than a shorter one. If this is true, it should take a measurable amount of time more for each extension. In order to calculate how long each extra extension took, we did a multiple linear regression, using the distance along the x axis participants had to extend the line. This analysis was significant, $r = .41$, $p < .01$. The analysis yielded the following formula: Response Time $= 8.21 + 1.28(1.2)$, where 8.21 seconds is the baseline time to read information from the graph and 1.28 is the amount of extra time required to extend the line each 1.2 cm distance required. This result supports our hypothesis that participants used spatial transformations, by indicating a systematic relationship between response time and the distance mentally traveled. As participants had to draw longer mental extensions to the graph, their response times systematically increased.

Finally, participants' self-reports indicate that they used a spatial strategy. After participants completed the tasks, we interviewed them about their strategies for performing each task. Participants unanimously said something like "I went straight up and over" in reporting how they performed the read-off task. For the near and far tasks, they all reported some variation on extending the line. Typical responses included, "I imagined where the line would go," "I estimated how far you think—, the angle the line is going to go," "You have to project the line with your eye and then go from the arrow to the middle and over to the y axis." These comments indicate that participants relied on the spatial characteristics of the graph to make inferences.

In summary, we have found that when people make inferences about simple graphs, their accuracy, response time, and self-report data suggest that they use spatial reasoning, in the form of spatial transformations. Given that current theories of graph comprehension provide no account of mechanisms by which people make such inferences, we propose that a comprehensive theory of graph comprehension should accommodate spatial reasoning, as indicated by these data.

References

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